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MEASURES OF RELATIVE POSITION AND DERIVED SCORES

The meaning of any absolute measure is limited, until and unless it is not explained in relative terms. Is the error of two inch in measuring the height a big error? This question can be answered only when it is known whether the height measured is of a child or of a building. The price of any commodity is meaningful only in reference to some norms, or in reference of the cost of similar objects or the money available for purchasing.

In the measurement of human traits, these norms are obtained from the groups to which this member belongs. It is impossible to think about the quantity of some trait in an individual, until and unless we know about that trait among normal individuals or we explain it in, reference to this trait in general individuals.

For example, if a child of 10 years of age is left in a group of children, who are younger to him in age, then he will seem to be more longer and matured, but if he is shiflted to such a group in which he is the youngest, then he will feel to be young and immatured. In this situation, the standard is changed, but in case of absolute measures like age, height, such changes are not projected in order to understand the behaviour of an individual, his relative position in the group, and the group behaviour have special signifiance.

Specially, the psychological and educational measures have no meaning, until and unless they are presented in reference to some norms. What does it mean that a student has obtained 30 marks? These marks may be very high or very low or average. Thus, absolute number do not provide any meaning itself.

In this chapter, such measures will be discussed, which clarify the relative postion of an individual. For example, what is the position of an individual who obtained 30 marks? How many individuals obtained more marks than this individual ?, etc.

PERCENTILE RANK

Percentile rank provides a relative rank in a distribution, which is based on a scale of 100. The percentile rank of a score, tells what percentage of scores lie below that particualar given percentile rank. If your percentile rank is 8, then it means that 8% scores lie below your rank. According to **Ary and Jacobs, "A percentile rank gives us the relative rank of a score within a distribution, based on a scole of 100." While Garrett says, "The percentile rank (PR) of an individual is the position on a scale of 100 to which the subject's score entitles him."**

Because the percentile rank indicates the position of a score in reference to ranks, thus it is an ordinal statistics. There is no sense of a rank, until and unless the total number of score are kown in a distribution. A score of 8^{th} percentile rank in a distribution of 200 scores is relatively low score, but securing 8^{th} percentile rank in a distribution of 10 score is relatively high score. **Percentile rank by employing a scale of 100, provides such an index of position, which is independent of number of scores in the distribution.**

Percentile rank has a universal meaning, which is not found in original scores. Any percentile rank near zero, means a low position in the group, similarly a PR near 50 means average position in the group and any PR near 100 always have a high postion in the group. To have a 96^{th} percenile rank in a class of statistics means very high position in the class. One should remember that percentile rank tells the position of an individual in a group, it is not an absolute value. In reference of beauty, a lady getting fourth percentile rank in Miss Universe Competition may be attractive, but in a population of prisoners, getting 96^{th} percentile rank for truthfulness, may not necessarily an honest individual. Therofore, in evaluting a percentile rank of an individual for comparing it is essential to know the nature of the group.

Generally, people fail to discriminate percentage and percentile. A student, responding 15 correct responses out of 20, actually responded ·75% correct responses, but one cannot tell his percentile rank, until and unless the mark of other individuals in the group are known. If 15 is a high score in the group, then the individual scoring 15 will obtain a high percantile rank. But, if most of other individuals obtained more than 15 marks, then the individual securing fifteen marks will get a low percentile rank. **Here, one should remember the first division is an index of high position but first percentile rank is an index of low position.**

CALCULATING PERCENTILE RANK

Percentile ranks can be computed from organized data, as well as from serially arrange scores. When scores are arrange in a serial order, then percentile rank (PR) can be computed from the following formula :

$$P.\,R. = 100 - \frac{100R - 50}{N} \qquad .....(1)$$

Where,

PR = Percentile Rank

R = Rank of Score

N = Total number of Scores.

Example 1. In a class of 20, students are ranked from 1 to 20. Find out the percentile ranks of students of obtaining ranks of 9, 13 and 16.

Solution—(a) The percentile rank of student getting 9^{th} rank will be :

$$\begin{aligned}
PR &= 100 - \frac{100R-50}{N} \\
&= 100 - \frac{100\times 9-50}{20} \\
&= 100 - \frac{900-50}{20} \\
&= 100 - \frac{850}{20} \\
&= 100 - 42{\cdot}50 \\
&= 57{\cdot}50 \\
&\text{or } \mathbf{57}
\end{aligned}$$

(b)

$$\begin{aligned}
PR &= 100 - \frac{100R-50}{20} \\
&= 100 - \frac{100\times 13-50}{20} \\
&= 100 - \frac{1300-50}{20} \\
&= 100 - \frac{1250}{20} \\
&= 100 - 62{\cdot}5 \\
&= \mathbf{37{\cdot}5} \\
\text{or } &= \mathbf{37}
\end{aligned}$$

(c)

$$\begin{aligned}
PR &= 100 - \frac{100\times 16-50}{20} \\
&= 100 - \frac{1600-50}{20}
\end{aligned}$$

$$= 100 - \frac{1550}{20}$$

$$= 100 - 77.50$$

$$= \mathbf{22{\cdot}50}$$

or **22**

Thus, the students getting 9, 13, 16 ranks will get a percentile ranks of 57, 37 and 22 respectively.

CALCULATION OF PERCENTILE RANK F ROM GROUPED DATA

When date is grouped then PR is calculated from the following formula :

$$PR = \frac{100}{N}\left[F - \frac{(X-L)\,fq}{i}\right] \qquad \text{......(2)}$$

Where :

X = The score for which PR is to be calculated.

L = Exact lower limit of the class interval in which X lies.

fq = Frequencies of the class interval in which X lies.

F = Cumulative frequencies below the class–interval in which X lies.

N = Total number of frequencies.

i = Size of the class–interval.

The following steps followed in calculating PR :

1. Find out the class interval in which the score falls, for which PR is sought.
2. Find out the exact lower limit of the class interval in which X lies.
3. Calculate the cumulative frequencies (F).
4. Divide the frequencies of desired class–interval by the size of class interval (i) to find out the rate of step deviation.
5. Multiply the difference of X and exact lower limit by the rate of step deviation.
6. Add this product to the cumulative frequencies below the class–interval, in which X lies.
7. Multiply this sum by 100.
8. The result of step seven now be divided by N. One can reverse the process of step 7 and 8, *i.e.*, first divide the sum by N and then multiply it by 100.

Example 2. Find out the percentile rank of an individual securing 28 marks from the distribution of scores given in table1.

TABLE 1

Class-interval	f	F
40-40	2	25
35-39	3	23
30-34	4	20
25-29	5	16
20-24	5	11
15-19	3	6
10-14	2	3
05-09	1	1

N = 25

Sulution—

$$PR = \frac{100}{N}\left\{F+\frac{(X-L)fq}{i}\right\}$$

$$= \frac{100}{25}\left\{11+\frac{(28-24\cdot5)5}{5}\right\}$$

$$= 4\left\{11+\frac{3\cdot5\times5}{5}\right\}$$

$$= 4\left(11+\frac{17\cdot5}{\cdot5}\right)$$

$$= 4\ (11 + 3\cdot5)$$

$$= 4\ (14\cdot5)$$

$$= \mathbf{58\cdot0}$$ **Ans.**

USES OF PERCENTILE RANK

Percentile ranks convert the raw scores in a form, which can easily be understood and which can easily be interpreted. They provide convenient base for comparison and interpretation. As percentile is a relative measure, it should be kept in mind the nature of individuals among whom the rank is given. A percentile rank indicates the position of an individual in a specific group only. The individual must have the knowledge of the nature and structure of the group, for a percentile rank to be meaningful.

Raw scores are not distributed equally among a distribution. Therefore, percentile rank does not project the equal differences on achievement scale. Percentile rank depends on the fact that how many scores fall below the score in question. If there are congregation of several scores near a score, then slight increase in the score would take it several steps upward, with the result percentile ranks will also increase. In case there are very few score near a score, then even significant increase in the score will not effect the perentile rank.

PERCENTILES AND CENTILES

The inverse of percentile rank is percentile. Whereas, the percentile rank of a particular score unit is the percentage of score falling below this point in the ordred series of scores, and the value of this point itself is the percentile corresponding to this percentile rank. Thus, the 80^{th} percentile is the point on the score scale, below which, 80 percent of scores fall. The percentile rank of this point itself is 80, but the particular value of this point itself is the 80^{th} percentile. **Blomes** and **Lindquist** define it as below :

"The x^{th} percentile of a given score distribution is the point on the score below which x percent of score fall."

Downie and **Heath** describe it as following :

"A centile or cenitle point is defined as a specific point in a distribution which has a given percent of the case below it."

Like percentile rank, percentile also form an "ordinal scale". It is important for student to understand the difference between percentile rank and percentile. The percentile rank of any score is the percentage of score, which in the whole series falls below it. As explained before, percentile ranks on the scale of raw scores is the corresponding point of percentile. In the words of **Ary** and **Jacobs**,

"The percentile rank is a point on the original score scale corresponding to the particular percentile."

On the other hand, percentile on a scale of raw scores is a point on a given distribution, below which, X percent scores fall. In computing percentile ranks, one starts with original score X; and determines how many scores fall below it. In case of percentile, the individual determines the point below which X percent scores falls.

Percentile or centile is depicted by P_x, in which X shows the specific percent. Thus, 90^{th} percentile is written as P_{90}.

There ae certain percentiles which have special importance and are designated by special names and symbols. The nine percentiles which divide the distribution in ten equal parts are known as **deciles**. Decile point are below which 10 percent scores fall, (*i.e.* the 10^{th} percentile P_{10}) is known as first decile. The second decile point is the one below which 20 percent score falls.

The three percentile points which divide the distribution in four equal parts are known as quartiles. The point below which 25 percent cases lie is P_{25} and is known as first or lower quartile and designated by symbol Q_1. The point below which 50 percent cases fall (P_{50}) is known as second or middle quartile and is symbolized as Q_2. The Q_3 is a point below which 75 percent cases lie. It is also known as upper quartile.

The percentile point which divides the distribution into two equal parts, below and above which 50 percent cases lie is known as median. The median is equal to fifth decile D_5 and second quartile Q_2. These special percentils are summarized in Table 2.

In defining P_x, no restriction was imposed on the value of X, except that it should lie between the limits of 0 to 100. Usually, we are interested in nine points, which divided the distribution in ten sets of scores and are called deciles. The 99 points, which divide the distribution in equal sets of scores are known as centiles.

It is important to note that percentiles are points on the score scale and are not, as sometimes erroneously thought, intervals along this scale.

TABLE 2

SPECIAL PERCENTILE POINTS

Name	Symbols	Percentile
First Decile	D_1	P_{10}
Second Decile	D_2	P_{20}
Third Decile	D_3	P_{30}
Fourth Decile	D_4	P_{40}
Fifth Decile	D_5	P_{50}
Sixth Decile	D_6	P_{60}
Seven Decile	D_7	P_{70}
Eight Decile	D_8	P_{80}
Nine Decile	D_9	P_{90}
Firster Lower Quertile	Q_1	P_{25}
Second or Middle Qurtile	$Q_2 = D_5 = Mdn$	P_{50}
Third or upper Quertile	Q_3	P_{75}
Median	$Mdn = D_5 = Q_2$	P_{50}

COMPUTATION OF PERCENTILES

When scores are unorganized or in raw form then percentiles are computed from the following formula :

$$P_x = \frac{X(N+1)^{th}}{100} \text{ item} \qquad \text{......(3)}$$

Where,

P_x = The percentile to be calculated

X = Percentile to be calculated

N = Number of Scores

Thus, $$P_{15} = \frac{15(N+1)^{th}}{100} \text{ item}$$

$$P_{20} = \frac{20(N+1)^{th}}{100} \text{ item}$$

$$P_{35} = \frac{35(N+1)^{th}}{100} \text{ item}$$

Example 3. Find out P_{85} and P_{55} of the following scores 10, 11, 12, 15, 17, 19, 20, 22, 28, 30.

Solution :

$$P_{85} = \frac{85(10+1)^{th}}{100} \text{ item}$$

$$= \left(\frac{85 \times 11}{100}\right)^{th} \text{ item}$$

$$= \frac{935}{100}^{th} \text{ item}$$

$$= 9{\cdot}35^{th} \text{ item}$$

$$= 28 + {\cdot}35 \times 2$$

$$= 28 + {\cdot}70$$

$$= \mathbf{28{\cdot}70}$$

$$P_{55} = \frac{55(10+1)^{th}}{100} \text{ item}$$

$$= \left(\frac{55 \times 11}{100}\right)^{th} \text{ item}$$

$$= \left(\frac{605}{100}\right)^{th} \text{ item}$$

$$= 6{\cdot}05^{th} \text{ item}$$

$$= 19 + {\cdot}05 \times 1$$

$$= 19 + {\cdot}05$$

$$= \mathbf{19{\cdot}05 \text{ Ans.}}$$

Formula for calculating percentile from the organized data is

$$P_X = L + \left(\frac{P_N \times F}{f_p}\right)^{th} i \qquad \text{.....(4)}$$

Where

L = Exact Lower limit of the class interval in which desired percentile lies.

P_N = Desired percentage of N

F = Cummulative frequencies below the class interval in which P_N lies.

f_p = Freqenciess of P_N class–interval

i = Size of the class interval

Example 4. Compute P_{87} and P_{17} from the data given in Table 3.

TABLE 3

Class-Interval	f	F
40–44	2	25
35–39	3	23
30–34	4	**20**
25–29	5	16
20–24	5	11
15–19	3	6
10–14	2	3
05–09	1	1
	N = 25	

Solution—

$$P_{87} = L + \left(\frac{P_N - F}{f_p}\right) i$$

In the question

$$P_N = \frac{25 \times 87}{100}$$

$$= \frac{87}{4}$$

$$= \mathbf{21{\cdot}75}$$

$L = 34{\cdot}5$
$F = 20$
$f_p = 3$
$i = 5$

Putting these values in the formula

$$P_{87} = 34{\cdot}5 + \left(\frac{21\text{-}{\cdot}75 - 20}{3}\right) 5$$

$$= 34{\cdot}5 + \frac{1{\cdot}75}{3} \times 5$$

$$= 34{\cdot}5 + \frac{8{\cdot}75}{3}$$

$$= 34{\cdot}5 + 2{\cdot}916$$

or $= 37{\cdot}416$ or **37·42 Ans.**

Similarly

$$P_{17} = 14{\cdot}5 + \left(\frac{4{\cdot}25 - 3}{5}\right) 5$$

$$= 14{\cdot}5 + \frac{1{\cdot}25}{3} \times 5$$

$$= 14{\cdot}5 + \frac{6{\cdot}25}{3}$$

$$= 14{\cdot}5 + 2{\cdot}08$$

$$= \mathbf{16{\cdot}58}$$

EXERCISES

1. Explain the importance of Relative Measures.
2. What is the difference between Percentile rank and Percentile score ?
3. In what situation Percentile rank is used ?
4. What are the important Percentiles and Deciles ?